

# CP violation and chiral symmetry breaking in hot and dense quark matter in presence of magnetic field

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We investigate chiral symmetry breaking and strong CP violation effects on the phase diagram of strongly interacting matter in presence of a constant magnetic field. The effect of magnetic field and strong CP violating term on the phase structure at finite temperature and density is studied within a three flavor Nambu-Jona-Lasinio (NJL) model including the Kobayashi-Maskawa-t'Hooft (KMT) determinant term. This is investigated using an explicit variational ansatz for ground state with quark anti-quark pairs leading to condensates both in scalar and pseudoscalar channels. Magnetic field enhances the condensate in both the channels. Inverse magnetic catalysis for CP transition at finite chemical potential is seen for zero temperature and for small magnetic fields.

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## I. INTRODUCTION

The study of Charge-Parity ( $CP$ ) violation in strong interaction is of immense importance in the context of early universe scenario[1, 2] as well as heavy ion collision experiments [3–5]. Strong interaction respects space time reflection symmetry to a very high degree. However, this is not a direct consequence of quantum chromodynamics (QCD). The existence of instanton configurations for QCD allows for a nontrivial topological term in the action, the so-called  $\theta$ -term of QCD Lagrangian given as

$$\mathcal{L}_\theta = \frac{\theta}{64\pi^2} g^2 F_{\mu\nu}^a \tilde{F}^{a\mu\nu}. \quad (1)$$

In the above,  $F_{\mu\nu}^a$  is the gluon field strength and  $\tilde{F}^{\mu\nu}$  being its dual. This term violates charge conjugation and parity unless  $\theta = 0$  and  $\pm\pi$ . However, experiments on neutron dipole moment set limit on the value of  $\theta$  as  $\theta < 0.7 \times 10^{-11}$  [6]. This smallness of the CP violation term or its complete absence is not understood completely though a possible explanation is given in terms of spontaneous breaking of a new symmetry the Peccei-Quinn symmetry[1]. In vacuum, i.e. for zero temperature and zero density, parity is preserved when  $\theta = 0$  exactly [7]. However, this is spontaneously broken at  $\theta = \pi$ [8]. This P violation, called the Dashen mechanism is essentially nonperturbative.

Even if CP is not violated for QCD vacuum, it is conceivable that it can be violated for QCD matter at finite temperature or density. In deed, it has been proposed that hot matter produced in heavy ion collision experiments can give rise to domains of meta stable states that violate CP locally [3]. Different experimental observables for detecting such a phase have been suggested [9]. Apart from producing high temperature, colliding nuclei also produce transient strong magnetic fields. A nonzero  $\theta$  leads to a deviation of left and right handed helicity quarks. As a consequence an electromagnetic current is generated along the magnetic field. Such a mechanism known as chiral magnetic effect (CME)[4] may explain the charge separation in the recent STAR results [5]. This makes the study of chiral symmetry breaking mechanism at finite  $\theta$  interesting at finite temperature and magnetic fields. On the other hand, in the context of cold and dense matter, compact stars can be strongly magnetized. The magnetars, which are strongly magnetized neutron stars may have strong magnetic fields of the order of  $10^{15}$ – $10^{16}$  gauss [10–16].

In the present work we intend to investigate how chiral transition is affected when the CP odd effects and a strong magnetic background is present for hot and dense matter. For this purpose, we adopt the three flavor Nambu Jona-Lasinio model as an effective theory for chiral transitions. The effect of axial anomaly and the strong CP violation here is included through the Kobayashi-Masakawa-t'Hooft determinant term that mimics the effects of nontrivial gauge field configuration. This term is also a function of  $\theta$  and is responsible for CP violation for non vanishing values of  $\theta$ . Such a term has been extensively studied earlier for NJL models with two flavors in Ref.s [17–19] for studying the effects of non zero  $\theta$  on the chiral transition. This has been further extended to the two flavor NJL model including Polyakov loop potential[20, 21]. We had earlier considered the effects of strong CP violation on chiral symmetry breaking for the realistic case of 2+1 flavor using the NJL model [22]. This was further extended in Ref.[21] to include the effects of Polyakov loop potential. In all these investigations the effects of magnetic field were not included. It is this question that we would like to investigate here.

Modification of the ground state of QCD for  $\theta = 0$  in connection with chiral symmetry breaking in presence of magnetic field has been investigated in different effective models- e.g. chiral perturbation theory[23], NJL model [24–27] as well as different quark models of hadrons. In various models it was seen that while magnetic field acts as a catalyser of chiral symmetry breaking, it was observed that medium effects can lead to inverse magnetic catalysis for the same particularly at finite chemical potentials. The effects of magnetic field as well as nonzero values for  $\theta$  has been considered in Ref.[28] within the chiral sigma model.

We organize the paper as follows. In section II, we consider the three flavor NJL model along with the CP violating theta dependent six fermion determinant interaction term that also breaks axial symmetry. Here, we also write down quark field operator expansions in presence of magnetic field. Using the same, we next consider a variational ground state with quark anti-quark pairs that is related to chiral symmetry breaking. The ansatz is taken general enough to include both scalar as well as pseudoscalar condensates. The pseudoscalar condensate takes nonzero values for finite values of  $\theta$ . In section III, we discuss the resulting phase diagram at finite temperature as well as finite density for different strengths of magnetic field and for various values of  $\theta$ . Finally, in section IV we summarize the results and conclusion with a possible outlook.

## II. NJL MODEL WITH CP VIOLATION AND AN ANSATZ FOR THE GROUND STATE

To describe the chiral phase structure of strong interactions including the CP violating effects, we use the 3-flavor NJL model along with the flavor mixing determinant term. The Lagrangian is given by

$$\mathcal{L} = \bar{\psi} (iD - m) \psi + G \sum_{A=0}^8 [(\bar{\psi} \lambda^A \psi)^2 + (\bar{\psi} i \gamma^5 \lambda^A \psi)^2] - K [e^{i\theta} \det\{\bar{\psi}(1 + \gamma^5)\psi\} + e^{-i\theta} \det\{\bar{\psi}(1 - \gamma^5)\psi\}], \quad (2)$$

where  $\psi^{i,a}$  denotes a quark field with color ‘ $a$ ’ ( $a = r, g, b$ ), and flavor ‘ $i$ ’ ( $i = u, d, s$ ), indices.  $D_\mu = \partial_\mu - iqA_\mu$  is the covariant derivative in the presence of external magnetic field  $\mathbf{B}$  which we assume to be constant and in the  $z$ -direction. Further, we choose the gauge such that the corresponding electromagnetic potential is given as  $A_\mu = (0, 0, Bx, 0)$ . The matrix of current quark masses is given by  $\hat{m} = \text{diag}_f(m_u, m_d, m_s)$  in the flavor space. We shall assume in the present investigation, isospin symmetry with  $m_u = m_d$ . Strictly speaking, when the electromagnetic effects are taken into account, the current quark masses of  $u$  and  $d$  quarks should not be the same due to the difference in their electrical charges. However, because of the smallness of the electromagnetic coupling, we shall ignore this tiny effect and continue with  $m_u = m_d$  in the present investigation of chiral symmetry breaking in strong interaction. In Eq. (2),  $\lambda^A$ ,  $A = 1, \dots, 8$  denotes the Gellman matrices acting in the flavor space and  $\lambda^0 = \sqrt{\frac{2}{3}} \mathbf{1}_f$ ,  $\mathbf{1}_f$  as the unit matrix in the flavor space. The four point interaction term  $\sim G$  is symmetric in  $SU(3)_V \times SU(3)_A \times U(1)_V \times U(1)_A$ . In contrast, the determinant term  $\sim K$ , which generates a six point interaction for the case of three flavors, breaks  $U(1)_A$  symmetry for vanishing  $\theta$  values. In the absence of the magnetic field and the mass term, the overall symmetry in the flavor space is  $SU(3)_V \times SU(3)_A \times U(1)_V$ . This spontaneously breaks to  $SU(3)_V \times U(1)_V$  implying the conservation of the baryon number and the flavor number. The current quark mass term introduces additional explicit breaking of chiral symmetry leading to partial conservation of the axial current. Due to the presence of magnetic field, on the other hand, the  $SU(3)_V$  symmetry in the flavor space reduces to  $SU(2)_V \times SU(2)_A$  since the  $u$  quark has different electric charge compared to  $d$  and  $s$  quarks [29]. The effect of CP violating topological term of Eq.(1) is simulated by the determinant term of Eq.(2) in the quark sector. This can be easily seen by taking the divergence of the flavor singlet axial current

$$\partial_\mu J_5^\mu = 2i\bar{\psi} m \gamma^5 \psi + 2iN_f K (e^{i\theta} \det\bar{\psi}(1 + \gamma^5)\psi - h.c.), \quad (3)$$

where,  $J_5^\mu = \bar{\psi} \gamma^\mu \gamma^5 \psi$  summed over all the flavors. This equation may be compared with the usual anomaly equation written in terms of the topological term for the gluon field arising from Eq.(1)

$$\partial_\mu J_5^\mu = 2i\bar{\psi} m \gamma^5 \psi + 2N_f \frac{\theta}{32\pi^2} g^2 F_{\mu\nu}^a \tilde{F}^{a\mu\nu}. \quad (4)$$

Thus the effect of gluon operator  $\frac{\theta}{32\pi^2} g^2 F_{\mu\nu}^a \tilde{F}^{a\mu\nu}$  is simulated by the imaginary part of the determinant term in the quark sector. Such a term can lead to formation of condensates in the pseudoscalar channel as we investigate in the following.

The quark field operators in presence of a constant magnetic field can be expanded in terms of creation and annihilation operators can be written as as[25]

$$\psi(\mathbf{x}) = \sum_n \sum_r \frac{1}{2\pi} \int d\mathbf{p}_x [q_r(n, \mathbf{p}_x) U_r(x, \mathbf{p}_x, n) + \tilde{q}_r(n, -\mathbf{p}_x) V_r(x, -\mathbf{p}_x, n)] e^{i\mathbf{p}_x \cdot \mathbf{x}_x}. \quad (5)$$

The sum over the integers  $n$  in the above expansion runs from 0 to infinity. In the above,  $\mathbf{p}_x \equiv (p_y, p_z)$ , and,  $r = \pm 1$  denotes the up and down spins. The two component quark and anti quark operators satisfy the quantum algebra

$$\{q_r(n, \mathbf{p}_x), q_{r'}^\dagger(n', \mathbf{p}_x')\} = \{\tilde{q}_r(n, \mathbf{p}_x), \tilde{q}_{r'}^\dagger(n', \mathbf{p}_x')\} = \delta_{rr'} \delta_{nn'} \delta(\mathbf{p}_x - \mathbf{p}_x'). \quad (6)$$

Further,  $U$  and  $V$  are the four component spinors for the quark and anti quarks respectively. For constant magnetic field they have been derived in Ref.[25]. These can be expressed in terms of the Hermite polynomials and are normalized as [25]

$$\int dx U_r(x, \mathbf{p}_x, n)^\dagger U_s(x, \mathbf{p}_x, m) = \delta_{n,m} \delta_{r,s} = \int dx V_r(x, \mathbf{p}_x, n)^\dagger V_s(x, \mathbf{p}_x, m) \quad (7)$$

With the operators defined for the quark fields in a constant magnetic field, we next consider an ansatz for the ground state as

$$|\Omega\rangle = U|0\rangle \quad (8)$$

where,  $U_q = U_I U_{II}$  is a unitary operator.  $U_I$  and  $U_{II}$  are unitary operators described in terms of quark-anti quark creation and annihilation operators. Explicitly they are given as

$$U_I = \exp \left( \sum_{n=0}^{\infty} \int d\mathbf{p}_x q_r^{i\dagger}(n, \mathbf{p}_x) a_{r,s}^i(n, p_z) f^i(n, \mathbf{p}_x) \tilde{q}_s^i(n, -\mathbf{p}_x) - h.c. \right) \quad (9)$$

where, we have retained flavor index  $i$  for the quark field operators. Further, in the above equation, the spin dependent structure  $a_{r,s}^i$  is given by [25]

$$a_{r,s}^i = \frac{1}{|\mathbf{p}^i|} \left[ -\sqrt{2n|q^i|B} \delta_{r,s} - ip_z \delta_{r,-s} \right] \quad (10)$$

with  $|\mathbf{p}^i| = \sqrt{p_z^2 + 2n|q^i|B}$  denoting the magnitude of the three momentum of the quark/anti quark of  $i$ -th flavor (with electric charge  $q^i$ ) in presence of a magnetic field. It is easy to show that,  $aa^\dagger = I$ , where  $I$  is an identity matrix in two dimensions. The ansatz functions  $f^i(n, p_z)$  are determined from the minimization of thermodynamic potential. This particular ansatz of Eq.(9) is a direct generalization of the ansatz considered earlier [30], for vacuum structure for chiral symmetry breaking to include the effects of magnetic field. Next, the unitary operator  $U_{II}$  is given as

$$U_{II} = \exp \left( \sum_{n=0}^{\infty} \int d\mathbf{p}_x q_r^{i\dagger}(n, \mathbf{p}_x) r g^i(n, \mathbf{p}_x) \tilde{q}_s^i(n, -\mathbf{p}_x) - h.c. \right) \quad (11)$$

The above construct of Eq.(8) is a generalization of the ground state structure in presence of a CP violating term same in Ref[22] to include the effects of a nonvanishing constant magnetic field as well. Clearly, in Eq.(8) the ansatz for the ground state has two arbitrary functions  $f^i$  and  $g^i$  which will be related to the condensates in the scalar and pseudoscalar channel respectively. The effect of temperature and density can also be implemented with such a nontrivial structure for the ground state using the formalism of thermo field dynamics [31, 32]. Here, the statistical average of an operator is given as an expectation value over a ‘thermal vacuum’. The methodology of TFD involves the doubling of the Hilbert space [31]. Explicitly, the ‘thermal vacuum’ is constructed from the ground state at zero temperature and density through a thermal Bogoliubov transformation given as

$$|\Omega(\beta, \mu)\rangle = \mathcal{U}_F |\Omega\rangle = e^{\mathcal{B}(\beta, \mu)^\dagger - \mathcal{B}(\beta, \mu)} |\Omega\rangle \quad (12)$$

with,

$$\mathcal{B}^\dagger(\beta, \mu) = \int \left[ \sum_{n=0}^{\infty} \int d\mathbf{k}_x q'_r(n, k_z)^\dagger \theta_-(k_z, n, \beta, \mu) \underline{q}'_r(n, k_z)^\dagger + \tilde{q}'_r(n, k_z) \theta_+(k_z, n, \beta, \mu) \underline{q}'_r(n, k_z) \right]. \quad (13)$$

In Eq.(13), the underlined operators are the operators in the extended Hilbert space associated with thermal doubling in TFD method, and, the ansatz functions  $\theta_\pm(n, k_z, \beta, \mu)$  are related to quark and anti quark distributions. All the functions,  $\phi^i$ ,  $g^i$  and  $\theta_\mp^i$  can be determined from extremization of the thermodynamic potential.

Realizing the fact that, the state given in Eq.(12) is obtained by successive Bogoliubov transformations, it is easy to calculate expectation values of different operators in terms of the ansatz functions. In particular, the scalar condensate for the  $i$ -th flavor can be written as

$$\begin{aligned} \langle \Omega(\beta, \mu) | \bar{\psi}^i \psi^i | \Omega(\beta, \mu) \rangle &= - \sum_{n=0}^{\infty} \frac{N_c |q^i| B \alpha_n}{(2\pi)^2} \int dp_z \cos \phi^i \cos 2g^i (1 - \sin^2 \theta_-^i - \sin^2 \theta_+^i) \\ &\equiv -I_s^i \end{aligned} \quad (14)$$

where,  $\alpha_n = (2 - \delta_{n,0})$  is the degeneracy factor of the  $n$ -th Landau level (all levels are doubly degenerate except the lowest Landau level). As we shall see later, the functions  $\sin^2 \theta_{\mp}$  will be related to the distribution functions for the quarks and anti quarks. Further, for later convenience, here we have introduced the function  $\phi^i \equiv \phi_0^i - 2f^i$  with  $\cot \phi_0^i = m^i / \epsilon_{ni}$ ,  $\epsilon^i = \sqrt{m^{i2} + 2n|q^i B|}$ . Similarly, the pseudoscalar condensate is given as

$$\begin{aligned} \langle \Omega(\beta, \mu) | \bar{\psi}^i \gamma_5 \psi^i | \Omega(\beta, \mu) \rangle &= - \sum_{n=0}^{\infty} \frac{N_c |q^i| B \alpha_n}{(2\pi)^2} \int dp_z \sin 2g^i (1 - \sin^2 \theta_-^i - \sin^2 \theta_+^i) \\ &\equiv -I_p^i \end{aligned} \quad (15)$$

Thus a non vanishing  $I_s^i$  will imply a chiral symmetry breaking phase while a non vanishing  $I_p^i$  will indicate a CP violating phase.

The energy density  $\mathcal{E}$  can be calculated by taking the expectation value of the Hamiltonian corresponding to the Lagrangian of Eq.(2) with respect to the state given in Eq.(12) as in Ref.s [22, 25]. The thermodynamic potential is then given by

$$\Omega = \mathcal{E} - \mu \rho - \frac{1}{\beta} s \quad (16)$$

In the above,  $\mu$  is the quark chemical potential and  $\rho$ , the total number density of the quarks is given by

$$\rho = \sum_{i=1}^3 \rho^i = \sum_i \langle \psi^{i\dagger} \psi^i \rangle = \sum_{i=1}^3 \sum_{n=0}^{\infty} \frac{N_c \alpha_n |q^i B|}{(2\pi)^2} \int dp_z [\sin^2 \theta_-^i - \sin^2 \theta_+^i]. \quad (17)$$

Finally, for the entropy density for the quarks we have [31]

$$s = - \sum_i \sum_n \frac{N_c \alpha_n |q^i B|}{(2\pi)^2} \int dp_z \{ (\sin^2 \theta_-^i \ln \sin^2 \theta_-^i + \cos^2 \theta_-^i \ln \cos^2 \theta_-^i) + (- \rightarrow +) \}. \quad (18)$$

Now the functional minimization of the thermodynamic potential  $\Omega$  with respect to the chiral condensate function  $\phi^i(p_z)$  and the pseudoscalar function  $g^i(p_z)$  leads to

$$\tan \phi^i = \frac{|\mathbf{p}^i|}{M_s^i} \quad \text{and} \quad \tan 2g^i = \frac{M_p^i}{\sqrt{M_s^{i2} + |\mathbf{p}^i|^2}}. \quad (19)$$

with  $|\mathbf{p}^i| = \sqrt{p_z^2 + 2n|q^i B|}$  and  $M_s^i$  and  $M_p^i$  are respectively the scalar and pseudoscalar contributions to the total mass for the  $i$ -th flavor. They are given by the solutions of the coupled gap equations

$$M_s^i = m^i + 4GI_s^i + K|\epsilon_{ijk}| \{ \cos \theta (I_s^j I_s^k - I_p^j I_p^k) - \sin \theta (I_s^j I_p^k + I_p^j I_s^k) \}, \quad (20)$$

$$M_p^i = 4GI_p^i - K|\epsilon_{ijk}| \{ \cos \theta (I_s^j I_p^k + I_p^j I_s^k) - \sin \theta (I_p^j I_p^k - I_s^j I_s^k) \}. \quad (21)$$

The above equations are actually *self consistent* equations for  $M_s^i$  and  $M_p^i$  because  $I_s^i$  and  $I_p^i$  are given in terms of  $M_s^i$  and  $M_p^i$  as in Eq.s (14,15) and Eq.(19). Finally, extremizing the thermodynamic potential with respect to the thermal function  $\theta_{\mp}$  leads to

$$\sin^2 \theta_{\pm}^{i,n} = \frac{1}{\exp(\beta(\omega_{i,n} \pm \mu^i)) + 1}, \quad (22)$$

where,  $\omega_{i,n} = \sqrt{M^{i2} + p_z^2 + 2n|q^i B|}$  is the excitation energy with the constituent quark mass  $M^i = \sqrt{M_s^{i2} + M_p^{i2}}$  arising from both scalar and pseudoscalar condensates.

Substituting the solution for the condensate function of Eq. (19) and the thermal function given in Eq.(22) back in Eq.s (14,15) yields respectively the scalar and pseudoscalar condensates as

$$I_s^i = \sum_{n=0}^{\infty} \frac{N_c |q^i| B \alpha_n}{(2\pi)^2} \int dp_z \left( \frac{M_s^i}{\omega_{i,n}} \right) (1 - \sin^2 \theta_-^i - \sin^2 \theta_+^i). \quad (23)$$

$$I_p^i = \sum_{n=0}^{\infty} \frac{N_c |q^i| B \alpha_n}{(2\pi)^2} \int dp_z \left( \frac{M_p^i}{\omega_{i,n}} \right) (1 - \sin^2 \theta_-^i - \sin^2 \theta_+^i). \quad (24)$$

Thus Eq.s(20),(21) and Eq.s(23),(24) define the self consistent mass gap equation for the  $i$ -th quark flavor. Using the solutions for the condensate function as well as the gap equations Eq.s(20,21), the thermodynamic potential given in Eq.(16) reduces to

$$\begin{aligned} \Omega = & -\frac{N_c}{4\pi^2} \sum_{n=0}^{\infty} \alpha_n \sum_i |q^i B| \int dp_z \omega_n^i \\ & -\frac{N_c}{4\pi^2 \beta} \sum_{n=0}^{n_{max}} \sum_i |q^i B| \int dp_z \left[ \ln \{1 + e^{-\beta(\omega_n^i - \mu)}\} + \ln \{1 + e^{-\beta(\omega_n^i + \mu)}\} \right] \\ & + 2G_s \sum_i \left[ I_s^{i^2} + I_p^{i^2} \right] + 4K \left[ \cos \theta \prod_{i=1}^3 I_s^i + \sin \theta \prod_{i=1}^3 I_p^i \right] \\ & - 2K |\epsilon_{ijk}| \left[ \cos \theta I_p^i I_p^j I_s^k + \sin \theta I_s^i I_s^j I_p^k \right] \end{aligned} \quad (25)$$

In the above the first term is the zero temperature and zero density term in presence of a constant magnetic field. the second term is the medium dependent term while the last two terms are the remaining interaction terms of the Lagrangian. For the CP violating parameter  $\theta \rightarrow 0$ , and the pseudoscalar density  $I_p^i \rightarrow 0$ , the thermodynamic potential reduces to the same as in Ref.[25]. The first term in Eq.(25) is ultraviolet divergent which is also transmitted to the gap equations Eq.s (20-21) through the integrals  $I_s^i$  and  $I_p^i$  in Eq.s (23-24) and need to be regularized to get any meaningful result. There have been different regularization schemes to tackle this divergence like Schwinger proper time method [34, 35], a smooth cut off [36]. We perform the regularization as in Ref.[25, 27] by adding and subtracting a zero field (vacuum) contribution which is also divergent. This makes the first term of Eq.(25) a rather appealing form of separating the zero field vacuum contribution that is divergent, and, a field dependent contribution which is finite. The divergent zero field vacuum contribution is then evaluated with a finite cutoff in the three momentum  $\Lambda$  as is usually done in NJL model without magnetic field. Thus, we write the first term of Eq.(25) as a sum of the vacuum contribution and the finite field contribution which is written in terms of Riemann-Hurwitz  $\zeta$  function as [25, 27]

$$\begin{aligned} & - \sum_{i=1}^3 \sum_{n=0}^{\infty} \frac{N_c \alpha_n |q^i B|}{(2\pi)^2} \int dp_z \sqrt{M^{i^2} + p_z^2 + 2n|q^i|B} \\ & = -\frac{2N_c}{(2\pi)^3} \sum_{i=1}^3 \int d\mathbf{p} \sqrt{|\mathbf{p}|^2 + M^{i^2}} \\ & - \frac{N_c}{2\pi^2} \sum_{i=1}^3 |q^i B|^2 \left[ \zeta'(-1, x^i) - \frac{1}{2}(x^{i^2} - x^i) \ln x^i + \frac{x^{i^2}}{4} \right], \end{aligned} \quad (26)$$

where, we have introduced the dimensionless quantity,  $x^i = \frac{M_s^{i^2} + M_p^{i^2}}{2|q^i B|} = \frac{M^{i^2}}{2|q^i B|}$ , i.e. the mass parameter in units of the magnetic field and  $\zeta'(-1, x) = d\zeta(z, x)/dz|_{z=1}$  is the derivative of the Riemann-Hurwitz zeta function. The zero field vacuum term in Eq.(26) can be calculated using a sharp cutoff as it is usually done in NJL model.

Using Eq.(26), the scalar and pseudoscalar condensates as given in Eq.s(23-24) can also be separated into a (divergent) vacuum term, a field dependent term which is finite and a medium dependent term which is also finite. Thus we can write the scalar condensate as

$$\begin{aligned}
I_s^i &\equiv -\langle \bar{\psi}^i \psi^i \rangle = \frac{2N_c}{(2\pi)^3} \int_{|\mathbf{p}| < \Lambda} d\mathbf{p} \frac{M_s^i}{\sqrt{\mathbf{p}^2 + M^i{}^2}} \\
&\quad + \frac{N_c M_s^i |q^i B|}{(2\pi)^2} \left[ x^i (1 - \ln x^i) + \ln \Gamma(x^i) + \frac{1}{2} \ln \frac{x^i}{2\pi} \right] \\
&\quad - \sum_{n=0}^{n_{max}} \frac{N_c |q^i| B \alpha_n}{(2\pi)^2} \int dp_z \frac{M_s^i}{\sqrt{M^i{}^2 + p_z^2 + 2n|q^i B|}} (\sin^2 \theta_-^i + \sin^2 \theta_+^i) \\
&= I_{s\,vac}^i + I_{s\,field}^{si} + I_{s\,med}^i.
\end{aligned} \tag{27}$$

The zero field vacuum contribution,  $I_{vac}^{si}$ , can be analytically calculated using a sharp momentum cutoff  $\Lambda$  and can be written as

$$I_{s\,vac}^i = \frac{N_c M_s^i}{2\pi^2} \left[ \Lambda \sqrt{\Lambda^2 + M^i{}^2} - M^i{}^2 \log \left\{ \frac{\Lambda + \sqrt{\Lambda^2 + M^i{}^2}}{M^i} \right\} \right]. \tag{28}$$

Further, since  $|\mathbf{p}| = \sqrt{p_z^2 + 2n|q^i B|}$ , the condition of a sharp cutoff in magnitude of three momentum leads to a finite number of Landau levels that are filled up till  $n = n_{max}$  which is given as  $n_{max} = \text{Int}[\frac{\Lambda^2}{2|q^i B|}]$  when  $p_z = 0$ . Further this condition also lead to a cutoff for  $|p_z|$  as  $\Lambda' = \sqrt{\Lambda^2 - 2n|q^i B|}$  for a given value of the Landau level  $n$ .

Similarly, we can write the pseudoscalar condensate as

$$\begin{aligned}
I_p^i &\equiv i\langle \bar{\psi}^i \gamma_5 \psi^i \rangle = \frac{2N_c}{(2\pi)^3} \int d\mathbf{p} \frac{M_p^i}{\sqrt{\mathbf{p}^2 + M^i{}^2}} \\
&\quad + \frac{N_c M_p^i |q^i B|}{(2\pi)^2} \left[ x^i (1 - \ln x^i) + \ln \Gamma(x^i) + \frac{1}{2} \ln \frac{x^i}{2\pi} \right] \\
&\quad - \sum_{n=0}^{n_{max}} \frac{N_c |q^i| B \alpha_n}{(2\pi)^2} \int dp_z \frac{M_p^i}{\sqrt{M^i{}^2 + p_z^2 + 2n|q^i B|}} (\sin^2 \theta_-^i + \sin^2 \theta_+^i) \\
&= I_{p\,vac}^i + I_{p\,field}^i + I_{p\,med}^i.
\end{aligned} \tag{29}$$

Here also the zero field vacuum contribution,  $I_{p\,vac}^i$ , can be analytically calculated using a sharp momentum cutoff  $\Lambda$  and can be written as

$$I_{p\,vac}^i = \frac{N_c M_p^i}{2\pi^2} \left[ \Lambda \sqrt{\Lambda^2 + M^i{}^2} - M^i{}^2 \log \left\{ \frac{\Lambda + \sqrt{\Lambda^2 + M^i{}^2}}{M^i} \right\} \right]. \tag{30}$$

Now, using Eq.(26), the thermodynamic potential can be rewritten as

$$\begin{aligned}
\Omega &= \Omega_{vac} + \Omega_{field} + \Omega_{med} \\
&\quad + 2G_s \sum_i \left[ I_s^{i2} + I_p^{i2} \right] + 4K \left[ \cos \theta \prod_{i=1}^3 I_s^i + \sin \theta \prod_{i=1}^3 I_p^i \right] \\
&\quad - 2K |\epsilon_{ijk}| \left[ \cos \theta I_p^i I_p^j I_s^k + \sin \theta I_s^i I_s^j I_p^k \right].
\end{aligned} \tag{31}$$

In the above,  $\Omega_{vac}$  is the vacuum contribution towards the thermodynamic potential and using a sharp cutoff, it can be analytically calculated as

$$\begin{aligned}
\Omega_{vac} &= -2N_c \sum_i \int_{|\mathbf{p}| < \Lambda} \frac{d\mathbf{p}}{(2\pi)^3} \sqrt{\mathbf{p}^2 + M^{i2}} \\
&\equiv -\frac{N_c}{8\pi^2} \sum_i \left[ (\Lambda^2 + M^{i2})^{1/2} (2\Lambda^2 + M^{i2}) - M^{i4} \log \frac{\Lambda + \sqrt{\Lambda^2 + M^{i2}}}{M^i} \right].
\end{aligned} \tag{32}$$

$\Omega_{field}$  is the field contribution to  $\Omega$  and is given by

$$\Omega_{field} = -\frac{N_c}{2\pi^2} \sum_i |q^i B|^2 \left[ \zeta'(-1, x^i) - \frac{1}{2}(x^{i2} - x^i) \ln x^i + \frac{x^{i2}}{4} \right], \tag{33}$$

where the derivative of the Riemann-Hurwitz zeta function  $\zeta(z, x)$  at  $z = -1$  is given by [33]

$$\zeta'(-1, x) = -\frac{1}{2}x \log x - \frac{1}{4}x^2 + \frac{1}{2}x^2 \log x + \frac{1}{12} \log x + x^2 \int_0^\infty \frac{2 \tan^{-1} y + y \log(1 + y^2)}{\exp(2\pi xy) - 1} dy. \tag{34}$$

Finally, the medium contribution  $\Omega_{med}$  towards the thermodynamic potential is given by

$$\Omega_{med} = - \sum_{n,i} \frac{N_c \alpha_n |q^i B|}{(2\pi)^2 \beta} \int dp_z \left[ \ln \left\{ 1 + e^{-\beta(\omega_n^i - \mu)} \right\} + \ln \left\{ 1 + e^{-\beta(\omega_n^i + \mu)} \right\} \right]. \tag{35}$$

The coupled mass gap equations Eq.(20), Eq.(21) and the thermodynamic potential Eq.(31) constitute the basis for our numerical results for various physical situations that we discuss in the following section.

### III. RESULTS AND DISCUSSIONS

The three flavor NJL model that we investigate here, has five parameters in total, namely the current quark masses for the non strange and strange quarks,  $m_q$  and  $m_s$ , the two couplings  $G_s$ ,  $K$  and the three-momentum cutoff  $\Lambda$ . We have chosen here  $\Lambda = 0.6023$  GeV,  $G_s \Lambda^2 = 1.835$ ,  $K \Lambda^5 = 12.36$ ,  $m_u = 5.5$  MeV  $m_d$  and  $m_s = 0.1407$  GeV as has been used in Ref.[38]. After choosing  $m^q = 5.5$  MeV, the remaining four parameters are fixed by fitting to the pion decay constant and the masses of pion, kaon and  $\eta'$ . With this set of parameters the mass of  $\eta$  is underestimated by about six percent and the constituent masses of the light quarks turn out to be  $M^{u,d} = 0.368$  GeV for u-d quarks, while the same for strange quark turns out as  $M^s = 0.549$  GeV, at zero temperature and zero density.

In the numerical calculations that follows, we have taken the quark chemical potential  $\mu$  to be same for all the three flavors. For a given values of  $T$ ,  $\mu$  and strength of magnetic field  $B$ , we first solve the gap equations (20) and (21) self consistently along with the condensates given in Eq.(27) and Eq.(29) with the parameters of the NJL model as above. Since we have assumed  $m_u = m_d$ , the equations actually represent four coupled equations for zero magnetic field : two corresponding to the scalar contributions towards the masses, i.e,  $M_s^u = M_s^d$  and  $M_s^s$  and two corresponding to the pseudoscalar contributions towards the masses, i.e,  $M_p^u = M_p^d$  and  $M_p^s$ . However, this degeneracy is lifted in presence of finite magnetic field. Thus Eq.s (20) and (21) actually represent six coupled mass gap equations– the contributions to the masses arising from the scalar and pseudoscalar condensates of each flavor. Once the solutions to these coupled equations for the masses and the condensates are found, they are then substituted in Eq.(31) to find the thermodynamic potential  $\Omega$ . In case of more than one solution to the gap equation, the solution with the minimum  $\Omega$  is chosen.

In our analysis, we have explored the behavior of scalar and pseudoscalar contributions to the quark mass with temperature, chemical potential and magnetic field for different values of  $\theta$ .

Let us first discuss the effect of magnetic field on the “vacuum” properties within the model. In Fig.1 we have plotted the constituent quark masses given as  $M^i = \sqrt{M_s^{i2} + M_p^{i2}}$  as a function of magnetic field for different representative values of  $\theta$ . Due to the different charges of the u and d quarks, the isospin symmetry is lost between the light quarks when an external magnetic field is applied to the system. The magnetic catalysis of dynamical generation of mass



is seen for all the quarks with the constituent quark masses increasing with magnetic field for all values of  $\theta$ . The constituent quark masses here, however, are generated from from quark anti quark condensates both in scalar and pseudoscalar channels for non zero values of  $\theta$ .

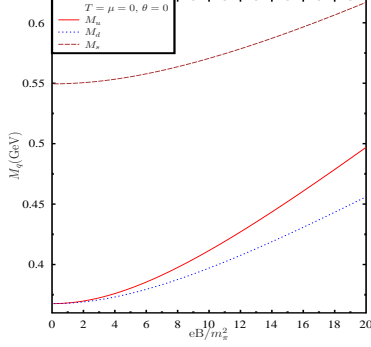


Fig. 1-a

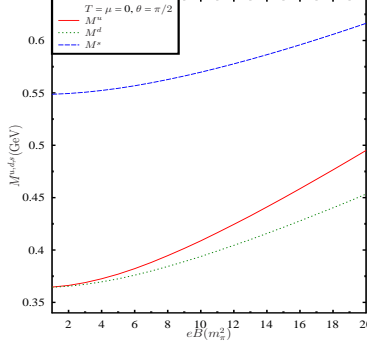


Fig. 1-b

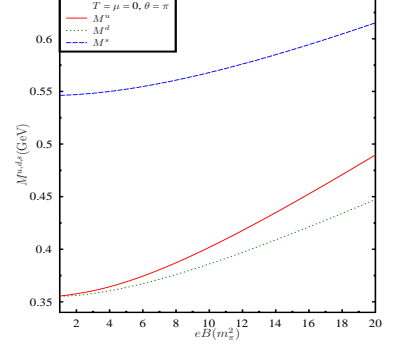


Fig. 1-c

FIG. 1. Constituent quark mass at  $T=0, \mu=0$  as a function of magnetic field for  $u$ - quark for  $\theta=0$ (Fig. 1-a),  $\theta=\pi/2$ (Fig. 1-b) and  $\theta=\pi$ (Fig.1-c)

In Fig.2 the condensates in scalar and pseudoscalar channel for  $u$ -quark is plotted as  $\theta$  increases for different magnetic fields. The condensates in the scalar and pseudoscalar channel vary in a complimentary manner so that the total constituent mass remains almost constant as  $\theta$  is varied. This behavior is also seen with increasing magnetic field along with the fact that the condensates in both the channels become larger in magnitude for larger magnetic field. The spontaneous CP violation is seen for  $\theta=\pi$  with two degenerate solution for the pseudoscalar condensate differing by a sign.

Fig.3 shows the variation of the effective potential with  $\theta$  for different strengths of magnetic fields. The effective potential shown here is normalized with respect to the effective potential at  $\theta=0$ . It is minimum when  $\theta=0$  which is consistent with the Vafa-Witten theorem. The behavior we see here is similar to what we observed without the magnetic field. The magnetic field only reduces the effective potential.

In Fig.(4) we show the temperature dependence of the total mass for the  $u$ - quark for different magnetic fields at zero chemical potential. For vanishing  $\theta$ , the total mass gets contribution only from the condensates in the scalar channel. Within the model, the chiral transition temperature increases with the magnetic field similar to several effective models as well as some lattice QCD models [40]. The general reason being, magnetic field enhances the condensates and hence requires higher temperatures to melt the condensate. As a result of the charge difference we obtain a higher transition temperature for  $u$ -quark than for  $d$ -quark with the difference becoming larger with larger magnetic fields. The chiral transition is a crossover due to finite current quark mass. However, in some of the recent lattice calculations, inverse magnetic catalysis near the critical temperature is observed leading to a reduction of the crossover transition temperature with magnetic field [41]. At sufficiently lower temperature, on the other hand, magnetic catalysis is observed in these lattice simulations with the condensates getting enhanced with magnetic field. Such an effect can be generated in an ad hoc manner by reducing the effective four fermion coupling by making it a function of temperature and magnetic field as in Ref.[42]. There have been other attempts to explain this by invoking paramagnetic contributions to the pressure with large magnetization[44]; magnetic inhibition due to neutral meson fluctuation [45] as well as a back reaction of the Polyakov loop which could be affected by magnetic field [46]. In the present work however, we shall continue to consider the consequences of the ansatz as in Eq.(8), to discuss the effect of the non vanishing  $\theta$  and magnetic field on the phase structure within the premises of NJL model.

As  $\theta$  is increased, the contribution to the mass from the pseudoscalar condensates also increases. We have also plotted the temperature dependence of the pseudoscalar component of  $u$ -quark mass arising from pseudoscalar condensates  $M_u^p$  for  $\theta=\pi/2$  and  $\theta=\pi$  in Fig. 5. As may be observed from Fig. 5, for  $\theta=\pi/2$  the CP transition is a crossover transition. At  $\theta=\pi$ , however, the CP transition is a second order transition with the pseudoscalar condensates smoothly vanishing at the critical temperature. Further, this CP restoration transition temperature increases with magnetic field. We should however note that what we considered here is the equilibrium uniform CP violating phase

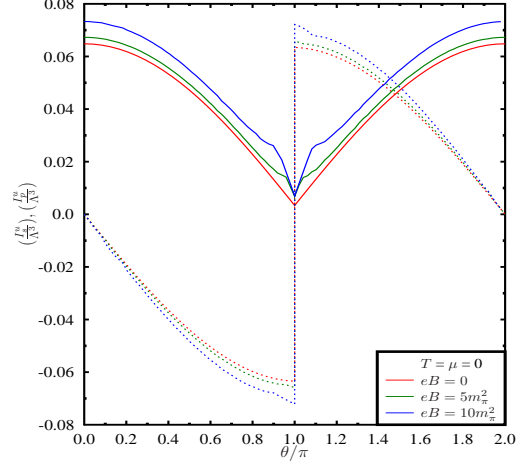


FIG. 2.  $\theta$  dependence of the order parameters for u -quarks for different magnetic fields. The solid lines correspond to condensates in the scalar channel while the dotted lines correspond to condensates in the pseudoscalar channel.  $\theta = \pi/2$ .

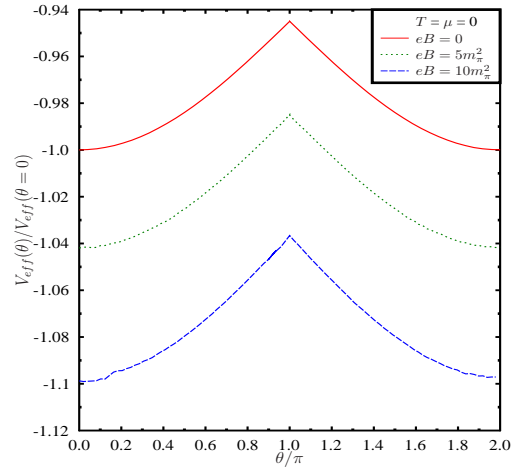


FIG. 3. Effective potential at  $T = \mu = 0$  as a function of  $\theta$  for different strengths of magnetic field..

structure induced by the determinant term. However, local parity violating phase can also arise due to fluctuations of topological charges induced through sphaleron configuration which are not exponentially suppressed [3]. On the other hand such domains can also arise due to non equilibrium situations depending upon the kinetics of the phase transition. Such CP odd domains can decay via CP odd processes and can have observable effects like chiral magnetic effect for non central heavy ion collisions[4] as well as possible excess in dilepton production for central collisions [37].

In Fig(6), we display the dependence of the constituent quark mass on quark chemical potential at zero temperature. The critical chemical potential where the total quark mass shows a discontinuity decreases with magnetic field. The transition is a first order one at zero temperature. Let us note that for  $\theta = 0$ , the entire mass arises from quark

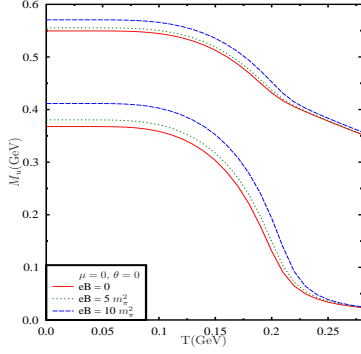


Fig. 4-a

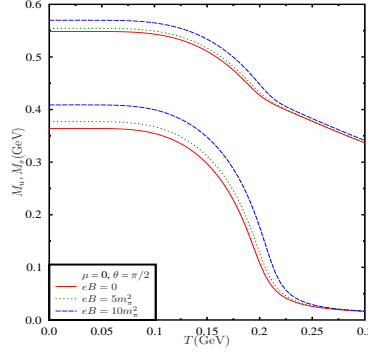


Fig. 4-b

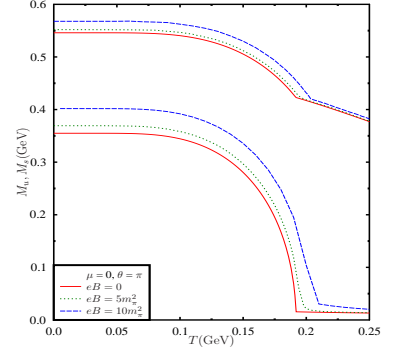


Fig. 4-c

FIG. 4. Temperature dependence of constituent quark masses for up and strange quarks for  $\theta = 0$ (Fig. 4-a),  $\theta = \pi/2$ (Fig. 4-b) and  $\theta = \pi$ (Fig.4-c) with different strengths of magnetic field. In each plot the lower curves are for the up quark mass variation while the upper curve shows temperature dependence of the strange quark mass .

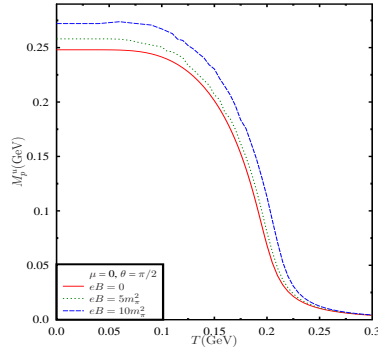


Fig. 5-a

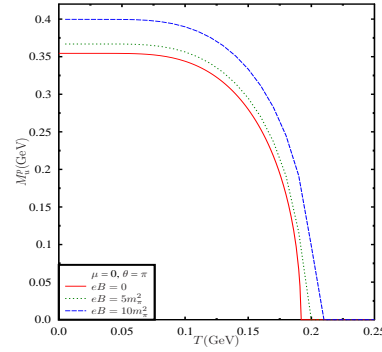


Fig. 5-b

FIG. 5. Temperature dependence of the pseudoscalar condensates for  $\theta = \pi/2$ (Fig. 5-a) and  $\theta = \pi$ (Fig.5-b) with different strengths of magnetic field.

condensates in the scalar channel apart from the current quark masses. This behavior of having lower critical chemical potential for higher magnetic field is the phenomenon of inverse magnetic catalysis of chiral symmetry breaking at finite chemical potential [25, 34]. For finite  $\theta$  however, the mass is generated by condensates in both scalar and pseudoscalar channels. It turns out that for  $\theta = \pi$ , at zero magnetic field the critical quark chemical potential is  $\mu_c \sim 545$  MeV with a first order transition . As magnetic field is increased,  $\mu_c$  decreases and is minimum at  $eB = 7m_\pi^2$  with  $\mu_c \sim 523$  MeV. As the magnetic field is increased further, the the critical chemical potential also increases and becomes about  $\mu_c \sim 560$  MeV for  $eB = 10m_\pi^2$ . Such a behavior of decrease of critical chemical potential for intermediate strengths of magnetic field and then increase for stronger fields is also observed in Ref. [43]. This inverse magnetic catalysis of CP transition at finite  $\theta$  can be understood in a manner similar to Ref.[43] discussed for chiral symmetry breaking in strong interaction. This can be easily done by analyzing the pseudoscalar mass gap equation Eq.(21). The analysis can be easily carried out for  $\theta = \pi$  for which we can approximate  $I_s^i \simeq 0$  as well as  $M_s^i \simeq 0$  for the light quarks. For large coupling, the solution to the gap equation is given by the  $\mu = 0$  solution. For

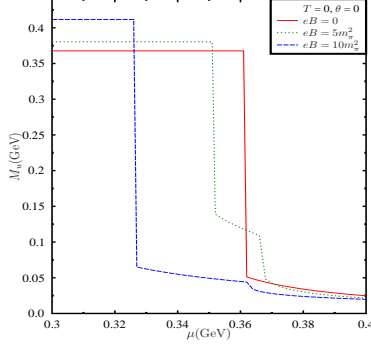


Fig. 6-a

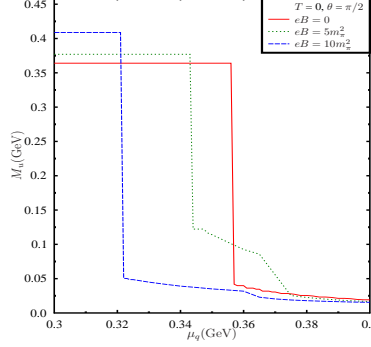


Fig. 6-b

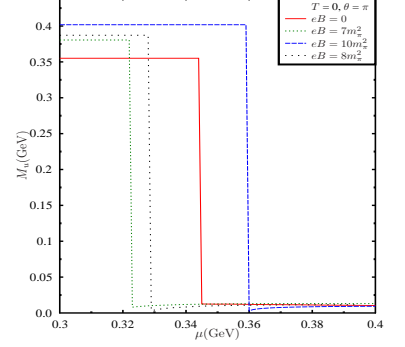


Fig. 6-c

FIG. 6. Up quark mass as a function of quark chemical potential at zero temperature for  $\theta = 0$ (Fig. 6-a),  $\theta = \pi/2$ (Fig. 6-b) and  $\theta = \pi$ (Fig.6-c) with different strengths of magnetic field.

nonzero but small magnetic fields, we can get the solution up to second order in magnetic field as

$$M_p^i \simeq M_0^i \left( 1 + \frac{G_s |q_i B|^2}{M_0^{i2} (1 - \frac{6G}{\pi^2} f(M_0^i, \Lambda))} \right), \quad (36)$$

where,  $f(M_0^i, \Lambda) = \Lambda^2 \left[ (1 + \hat{M}_0^{i2}) - \hat{M}_0^{i2} \log(\frac{1 + \hat{M}_0^{i2}/2}{\hat{M}_0^i}) \right]$ ;  $\hat{M}_0^i = M_0^i/\Lambda$  and,  $M_0^i$  is the solution of the pseudoscalar mass gap equation Eq.(21) with  $\mu = 0 = B$ . One can substitute this solution in the thermodynamic potential and subtract out  $M_p^i = 0$  free energy density. In the limit of nonzero but small magnetic field, the difference in thermodynamic potential is given as

$$\Delta\Omega \simeq -\frac{3}{8\pi^2} \sum_i M_0^{i2} \Lambda^2 \left( 1 - \frac{\pi^2}{3G\Lambda^2} \right) + \frac{3}{4\pi^2} |q_i B| \mu^2 - \frac{3}{4\pi^2} |q_i B|^2 \left( 1 + \log \frac{M_0^{i2}}{2|q_i B|} \right) \quad (37)$$

In the above, the first term is the vacuum ( $T=0, B=0, \mu=0$ ) contribution to the free energy difference. The term linear in the magnetic field correspond to the free energy cost to form a quark anti quark pair in the pseudoscalar channel at finite  $\mu$  which, also depends upon the magnetic field along with the chemical potential. The last term is the gain in thermodynamic potential due to condensation which is quadratic in magnetic field strength. Therefore, as we turn on the magnetic field, and start from broken phase with  $\Delta\Omega < 0$ , for small field it can make  $\Delta\Omega$  positive with the symmetry restored. However, as the field strength is increased, the quadratic term starts dominating and symmetry broken phase is preferred again and thus the critical chemical potential will increase with magnetic field. The behavior of the critical chemical potential with magnetic field is reflected in Fig.6 for  $\theta = \pi$  case.

#### IV. SUMMARY

In the present work, we have focussed on the effect of  $\theta$ - vacuum on the chiral transition for hot and dense matter in the presence of magnetic field. The effect of CP violating  $\theta$ - term in QCD is incorporated through a  $\theta$  dependent flavor mixing determinant interaction within a 3- flavor NJL model Lagrangian. The methodology uses an explicit variation construct for the ground state in terms of quark anti-quark pairing, instead of performing a chiral rotation of quark fields[20, 21]. The ansatz function in the variational construct for the ground state are determined from the minimization of the thermodynamic potential solving self consistent gap equations for the condensates in the scalar as well as the pseudoscalar condensates.

For non vanishing  $\theta$ , the constituent quark masses arise from quark anti quark condensates both in scalar and pseudoscalar channels. With increasing values of CP violating parameter  $\theta$ , the pseudoscalar condensates increase

and become maximum in magnitude at  $\theta = \pi$ . On the other hand, while the condensate in the scalar channel decrease with  $\theta$  and almost vanish for  $\theta = \pi$  but for the current quark mass contribution. The condensate in the two channels vary in a complimentary way such that constituent quark mass remains almost constant with  $\theta$  variation. Magnetic field enhances the condensates in *both* the channels and breaks the isospin symmetry of the light quarks.

The effective potential as a function of  $\theta$  shows the minimum at  $\theta = 0$  with cusp at  $\theta = \pi$  consistent with the Vafa-Witten theorem. Introduction of magnetic field does not change this behavior. It only reduces the magnitude of the effective potential.

At vanishing chemical potential, with temperature, the condensates in both the channel decrease. The CP transition is a second order transition at around  $T_c = 200 MeV$ . With magnetic field, this CP transition still remains second order with the transition temperature increasing as magnetic field strength is increased similar to the magnetic catalysis of chiral symmetry breaking at finite temperature. This high temperature restoration of CP is expected as the instanton effects responsible for CP violating phase become suppressed exponentially at high temperatures [39].

At finite chemical potential however, the CP transition is a first order transition. Further, inverse magnetic catalysis for the CP transition is observed at finite chemical potential at zero temperature i.e. the corresponding critical chemical potential decrease with magnetic fields for small magnetic fields. Possibility of a first order phase transition can lead to formation of CP-odd meta stable domains which could be of relevance for heavy ion collisions at the Facility for Anti proton and Ion Research (FAIR) as well as at Nucleotron based Ion Collider facility (NICA) at Dubna. However, it ought to be mentioned that for the application to heavy ion collision, it is crucial to include the non equilibrium dynamics of formation of domains, which will provide the relevant time scales and also provide informations on possibility of measuring the effects arising from the formation of such CP odd domains[47].

We have considered here quark-anti quark pairing in our ansatz for the ground state which is homogeneous with zero total momentum as in Eq.(8). However, it is possible that the condensate could be spatially non-homogeneous with a net total momentum [49–51] or for very strong fields could be non isotropic with vector condensation[48]. Further, one could include the effect of deconfinement transition by generalizing the present model to Polyakov loop NJL models for three flavors to investigate the inter relationship of deconfinement and the chiral transition[52] as well as CP violation[21] in presence of strong fields for the three flavor case considered here. This will be particularly important for finite temperature and low baryon densities. On the other hand, at finite density and small temperatures, the ansatz can be generalized to include the diquark condensates in presence of magnetic field [53–55]. Some of these calculations are in progress and will be reported elsewhere.

## ACKNOWLEDGMENTS

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